Technical Comments

Comment on "Wall Layer of Plane Turbulent Wall Jets without Pressure Gradients"

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HUBBARTT and Neale[1] have shown that the "wake component," or deviation from the law of the wall, in a wall jet in still air is well represented by

$$\Delta U/u_\tau = -33.3\ \mathrm{erf}(0.0652\ y/\delta_m) \qquad (1)$$

where δ_m is the value of y at which U is a maximum. The purpose of this comment is to point out that for $y << \delta_m$ Eq. (1) agrees very well with the wake component obtained by integrating the mixing length formula[2]

$$\partial U/\partial y = (\tau/\rho)^{1/2}/Ky \qquad (2)$$

with $\tau = \tau_w\ (1 - 2\ y/\delta_m)$. This variation of τ seems to be a good approximation outside the viscous sublayer; it was found in Ref. 3 and by later authors that $\tau \approx -\tau_w$ at $y = \delta_m$. The full integral of Eq. (2) is given in Ref. 2: let us merely note that the leading term for $\Delta U/u_\tau$ is $y(\partial\tau/\partial y)/(2K\tau_w)$ where K is von Kârman's constant, taken as 0.41 by Coles. Thus Eq. (2) gives

$$\Delta U/u_\tau = -2.44\,y/\delta_m \qquad (3)$$

while near the wall Eq. (1), as obtained from Fig. 3 of Ref. 1, is closely equal to

$$\Delta U/u_\tau = -2.38\ y/\delta_m \qquad (4)$$

The comparison cannot be extended to wall jets below a moving stream because $\partial\tau/\partial y$ is not known.

As usual, Eqs. (2) and (3) are expected to be valid only for y rather smaller than $\delta_m/2$ whereas the wholly-empirical form Eq. (1) appears to be valid at least as far as $y = \delta_{1/2}$ which cannot be explained by the wall-layer form of the mixing length formula. The main value of the comparison is the additional support it gives for Eq. (2) in regions of strong negative shear-stress gradient where its validity is sometimes questioned. For example, in Ref. 4, Vol. 1, p. 31, D. Coles argues that ΔU should be zero in the inner layer of a flow with $\partial\tau/\partial y < 0$, partly on the evidence of the data of Run 6300 of Ref. 4, Vol. 2, p. 504. However a typical value of $\Delta U/u_\tau$ predicted by Eq. (2) in Run 6300 is only −0.15; Hubbartt and Neale's data, with $\Delta U/u_\tau$ of the order of −2.0, provide more definite evidence, and this evidence supports Eq. (2) which is used directly or indirectly in many calculation methods for turbulent wall flows.

Received May 8, 1972.
Index category: Boundary Layers and Convective Heat Transfer—Turbulent.
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References

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[2]Townsend, A. A., "Equilibrium Layers and Wall Turbulence," *Journal of Fluid Mechanics*, Vol. 11, Aug. 1961, pp. 97–120.
[3]Bradshaw, P. and Gee, M. T., "Turbulent Wall Jets with and without an External Stream," Rept. and Memo 3152, 1960, Aeronautical Research Council, London, England.
[4]Kline, S. J., Morkovin, M. V., Sovran, G., and Cockrell, D. J. (editors), *Proceedings on the Computation of Turbulent Boundary Layers-1968 AFOSR-IFP-Stanford Conference*, Stanford Univ., Stanford, Calif., 1969.

Comparison of Geometric and Response-Feedback Approaches to Aircraft Lateral-Directional Decoupling

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AN ARTICLE by Eugene M. Cliff and Frederick H. Lutze,[1] is critical of the response-feedback approach to the same problem presented in Ref. 2. The purpose of this article is to clarify apparent misunderstandings of the response-feedback technique and compare it to the geometric-decoupling approach for determining the desired handling qualities solution.

Reference 1 refers to the approach taken in Ref. 2 as a "model-following" approach. We feel a more correct title would be "response-feedback" approach. This is indeed a small point since definitions vary. We prefer to define a model-following system as one in which the model is explicitly programmed in the system and followed on-line by the plant or simulator vehicle in this case. In Ref. 2, a model is defined which is only implicitly followed.

The intent of the study in Ref. 2 was to decouple a T-33 by matching the responses of a model of the following form

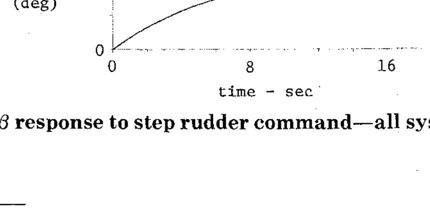

Fig. 1 β **response to step rudder command—all systems.**

Received January 30, 1973; revision received August 13, 1973.
Index category: Aircraft Handling, Stability and Control.
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